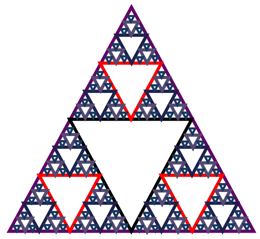
7.1 Isometric vs. Similar Figures

- **ISOMETRY** is a transformation that preserves side lengths and angle measurements. Examples:
  Rotation, Reflection or a composite of them
- The result (image) and the original are called **ISOMETRIC FIGURES** or **CONGRUENT FIGURES** (notation: $\cong$)
- **SIMILITUDE** is a transformation that results in a similar figure, but bigger/smaller in size. Example: Dilatation
- The result (image) is **SIMILAR** (notation: $\sim$) to the original: angles are congruent (to preserve the shape), but sides are proportional.

Translation,

The Sierpinski triangle is made entirely of **SIMILAR** triangles.



All of the triangles in this shape are SIMILAR because…

(1) All of the triangles contain the same angles.

(2) All of the side lengths of a triangle are <u>scaled</u> down by <u>the same</u> ratio of similarity k

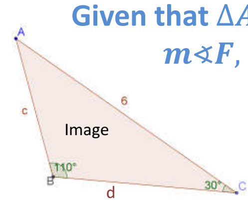
Ratio of similarity (scale factor): k

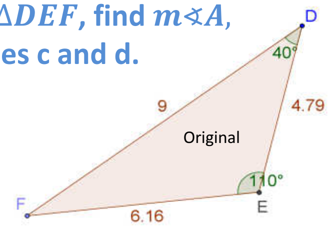
The ratio of similarity (scale factor) is

$$k = \frac{side\ length\ from\ the\ image}{side\ length\ from\ the\ original\ figure}$$

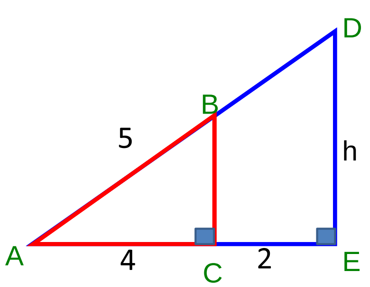
$$k = \frac{l_{new}}{l_{old}} = \frac{s'}{s}$$

Given that $\Delta ABC \sim \Delta DEF$, find $m\measuredangle A$, $m\measuredangle F$, and sides c and d.





Ex I: Finding an unknown side given 2 similar triangles



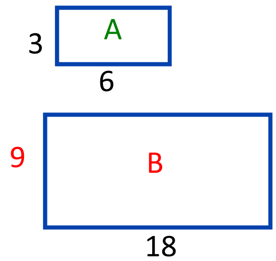
Practice: p. 216 # 1-5

7.1-B- Ratios of perimeters and areas

k	k^2	k^3
2		
	25	
		27
$\frac{2}{7}$		
	$\frac{9}{4}$	
		$\frac{8}{27}$

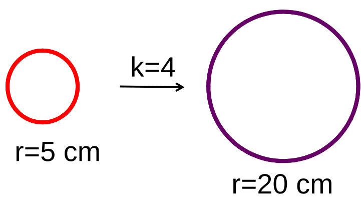
Understanding the perimeter ratio of two similar rectangles

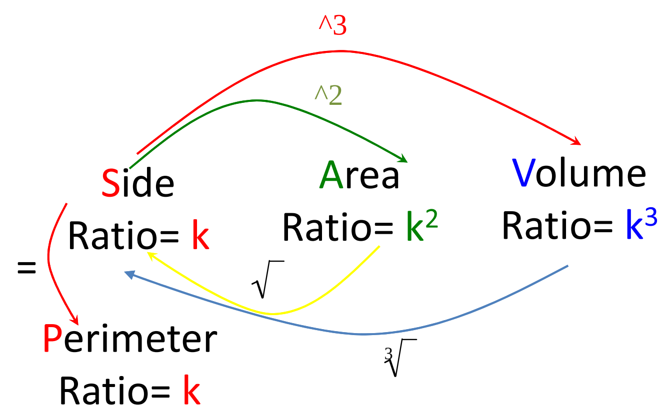
Ex 1: Figures A and B are similar



Understanding the area ratio of two similar circles

Ex 2:

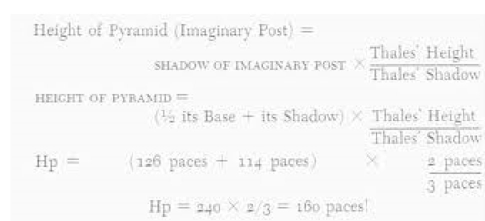




Application of Similar Triangles

The 3 great pyramids of Egypt are Khufo, Khafre, Menkaure

Their heights were unknown for over 2000 years, until about 600 BC, when Thales of Miletus, a Greek Mathematician calculated it.



Height of Pyramid (Imaginary Post) = SHADOW OF IMAGINARY POST × Thales' Height / Thales' Shadow

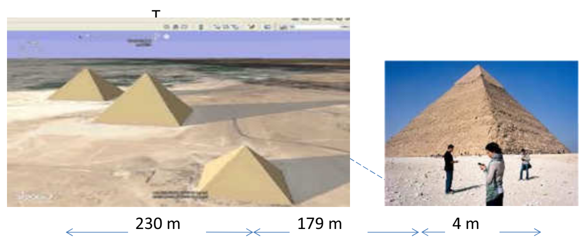
HEIGHT OF PYRAMID = (½ its Base + its Shadow) × Thales' Height / Thales' Shadow

Hp = (126 paces + 114 paces) × 2 paces / 3 paces

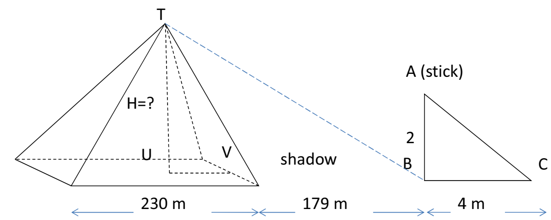
Hp = 240 × 2/3 = 160 paces!

(In his days they were measuring in cubits instead of m; 1cubit = 44.16 cm which is about 1 arm length)

He measured the length of the base and the length of the shadow. He then placed a 2 m stick at the end of the shadow and measured its shadow, it was 4 m long.



Since the sun creates equal angles on the ground, we have similar triangles: $\Delta ABC \sim \Delta TUB$;



Since the sun creates equal angles on the ground, we have similar triangles: $\Delta ABC \sim \Delta TUB$;

UV = 230/2 = 115;   so UB = 115 + 179 = 294m

Practice: page 218 # 6-10

7.2 Similar Solids and Ratios

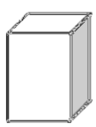
- Two solids are **isometric** if all corresponding angles and edges are congruent.
- Two solids are **similar** if all corresponding angles are congruent, and corresponding edges are proportional.
- Recall:
  - If the ratio between 2 similar solids is k ($\frac{s'}{s} = k$)
  - Then the ratio of Areas is k^2 ($\frac{A'}{A} = (\frac{s'}{s})^2 = k^2$)
  - And the ratio of Volumes is k^3 ($\frac{V'}{V} = (\frac{s'}{s})^3 = k^3$)

Steps to solve a problem:

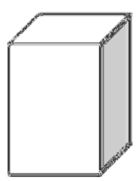
- First step is: find k then find k^2 and/or k^3
- to find a missing side:
  - write the ratio of sides = k ; then cross multiply
- to find a missing area:
  - write the ratio of areas = k^2 ; then cross multiply
- to find a missing volume:
  - write the ratio of volumes = k^3 ;then cross multiply
- **Note:** Keep the image always on top in your ratios

Ex 1: find the volume of the bigger prism

K = $\frac{3}{2}$



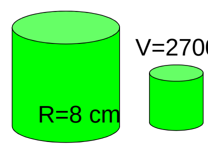
V = 16 cm^3



V = ?

K (sides)	k^2 (areas)	k^3 (volumes)
$\frac{3}{2}$ (given)	$\frac{9}{4}$	$\frac{27}{8}$

Ex 2: Determine the area of the base of the small cylinder if the two cylinders are similar.



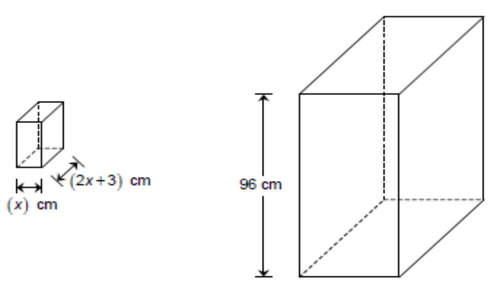
V=27000π cm^3

V=64000π cm^3

Shown below are two similar right prisms, each with a rectangular base.

The lengths, in centimetres, of the sides of the base of the smaller prism can be represented by the monomial (x) and the binomial $(2x+3)$ respectively. In this case, the volume of the smaller prism, in cm^3, is represented by the polynomial $(6x^3+9x^2)$.

The height of the larger prism is 96 cm. The total area of the larger prism is 64 times the area of the smaller prism.



What is the numerical volume of the larger prism?

Practice:
p. 221 # 1,2
p.222 # 10-15